



# Semi-Inclusive Deep Inelastic Scattering: nonperturbative and perturbative regimes

---

**A. Prokudin\*, M. Anselmino, M. Boglione, and C. Turk**

*Dipartimento di Fisica Teorica, Università di Torino and*

*INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy*

*E-mails: [prokudin@to.infn.it](mailto:prokudin@to.infn.it), [anselmino@to.infn.it](mailto:anselmino@to.infn.it),  
[boglione@to.infn.it](mailto:boglione@to.infn.it), [turk@to.infn.it](mailto:turk@to.infn.it)*

We consider the azimuthal and  $P_T$  dependence of hadrons produced in unpolarized Semi-Inclusive Deep Inelastic Scattering (SIDIS) processes, within the factorized QCD parton model. It is shown that at small  $P_T$  values,  $P_T \lesssim 1$  GeV/ $c$ , lowest order contributions, coupled to unintegrated (Transverse Momentum Dependent) quark distribution and fragmentation functions, describe all data. At larger  $P_T$  values,  $P_T \gtrsim 1$  GeV/ $c$ , the usual pQCD higher order collinear contributions dominate. Having explained the full  $P_T$  range of available data, we give new detailed predictions concerning the azimuthal and  $P_T$  dependence of hadrons which could be measured in ongoing or planned experiments by HERMES, COMPASS and JLab collaborations.

*DIFFRACTION 2006 - International Workshop on Diffraction in High-Energy Physics  
September 5-10 2006  
Adamantas, Milos island, Greece*

---

\*Speaker.

## 1. Introduction

In Ref. [1] a comprehensive analysis of Semi-Inclusive Deep Inelastic Scattering (SIDIS) processes within a factorized QCD parton model at  $\mathcal{O}(\alpha_s^0)$  was performed in a kinematical scheme in which the intrinsic transverse momenta of the quarks inside the initial proton ( $k_\perp$ ) and of the final detected hadron with respect to the fragmenting quark ( $p_\perp$ ) were fully taken into account. The dependence of the unpolarized cross section on the azimuthal angle  $\phi_h$  between the leptonic and the hadron production plane (Cahn effect [2]) was compared to the available experimental data, and used to estimate the average values of  $\langle k_\perp^2 \rangle$  and  $\langle p_\perp^2 \rangle$ .

In Ref. [1] the main emphasis, following the original idea of Cahn, was on the role of the parton intrinsic motion, with the use of unintegrated quark distribution and fragmentation functions. That applies to large  $Q^2$  processes, in a kinematical regime in which  $P_T \simeq \Lambda_{\text{QCD}} \simeq k_\perp$ , where  $P_T = |P_T|$  is the magnitude of the final hadron transverse momentum. In this region QCD factorization with unintegrated distributions holds [3] and lowest order QED elementary processes,  $\ell q \rightarrow \ell q$ , are dominating: the soft  $P_T$  of the detected hadron is mainly originating from quark intrinsic motion [4, 5, 6], rather than from higher order pQCD interactions, which, instead, would dominantly produce large  $P_T$  hadrons [7, 8, 9, 10, 11].

In this paper (see Ref.[12] for details) we start by showing that a complete agreement with data in the full range of  $P_T$  can be achieved; for  $P_T \lesssim 1$  GeV/c we follow the approach of Ref. [1] –  $P_T$  originated by the intrinsic  $k_\perp$  and  $p_\perp$  with  $\mathcal{O}(\alpha_s^0)$  partonic interaction – while in the range of  $P_T \gtrsim 1$  GeV/c we add the pQCD contributions – collinear partonic configurations with higher order [up to  $\mathcal{O}(\alpha_s^2)$ ] partonic interactions which generate the large  $P_T$ . We shall see that indeed most available data can be explained; the intrinsic  $k_\perp$  contributions work well at small  $P_T \lesssim 1$  GeV/c and fail above that, while the higher order pQCD collinear contributions explain well the large  $P_T \gtrsim 1$  GeV/c data and fail, or are not even applicable, below that. The two contributions match in the overlapping region,  $P_T \sim 1$  GeV/c, where it might be difficult to disentangle one from the other, as they describe the same physics. Infact parton intrinsic motions originate not only from confinement, but also from soft gluon emission, which, due to QCD helicity conservation, cannot be strictly collinear. Similar studies, concerning single transverse-spin asymmetries in Drell-Yan and SIDIS processes, with separate contributions – TMD quark distributions and higher-twist quark gluon correlations – in separate regions, have recently been published [13, 14].

## 2. Kinematics and Cross Sections

We consider SIDIS processes  $\ell p \rightarrow \ell h X$  in the  $\gamma^* p$  c.m. frame. The photon and the proton collide along the  $z$  axis with momenta  $q$  and  $P$  respectively; the leptonic plane coincides with the  $x$ - $z$  plane. We adopt the usual SIDIS variables (neglecting all masses):

$$\begin{aligned} s &= (P + \ell)^2 & (P + q)^2 &= W^2 = \frac{1 - x_{Bj}}{x_{Bj}} Q^2 & q^2 &= -Q^2 \\ x_{Bj} &= \frac{Q^2}{2P \cdot q} = \frac{Q^2}{W^2 + Q^2} & y &= \frac{P \cdot q}{P \cdot \ell} = \frac{Q^2}{x_{Bj} s} & z_h &= \frac{P \cdot P_h}{P \cdot q}. \end{aligned} \quad (2.1)$$

The SIDIS differential cross section can schematically be written in terms of a perturbative expansion in orders of  $\alpha_s$  as follows

$$d\sigma = \alpha_s^0 d\sigma_0 + \alpha_s^1 d\sigma_1 + \alpha_s^2 d\sigma_2 + \dots, \quad (2.2)$$

where  $d\sigma$  is a short hand notation to indicate  $\frac{d^5\sigma^{\ell p \rightarrow \ell h X}}{dx_{Bj} dy dz_h d^2P_T} = x_{Bj} s \frac{d^5\sigma^{\ell p \rightarrow \ell h X}}{dx_{Bj} dQ^2 dz_h d^2P_T}$ .

Lowest order cross section reads:

$$\begin{aligned} \frac{d^5\sigma_0^{\ell p \rightarrow \ell h X}}{dx_{Bj} dQ^2 dz_h d^2P_T} &= \sum_q \int d^2k_\perp f_q(x, k_\perp) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} J \frac{z}{z_h} D_q^h(z, p_\perp) \\ &= \sum_q e_q^2 \int d^2k_\perp f_q(x, k_\perp) \frac{2\pi\alpha^2}{x_{Bj}^2 s^2} \frac{\hat{s}^2 + \hat{u}^2}{Q^4} D_q^h(z, p_\perp) \frac{z}{z_h} \frac{x_{Bj}}{x} \left(1 + \frac{x_{Bj}}{x} \frac{k_\perp^2}{Q^2}\right)^{-1} \end{aligned} \quad (2.3)$$

we assume the usual  $x, k_\perp$  or  $z, p_\perp$  factorization, with a gaussian  $k_\perp$  and  $p_\perp$  dependence:

$$f_q(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle} \quad D_q^h(z, p_\perp) = D_q^h(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-p_\perp^2 / \langle p_\perp^2 \rangle}, \quad (2.4)$$

One introduces the parton variables  $x'$  and  $z'$ , defined similarly to the hadronic variables  $x_{Bj}$  and  $z_h$ ,

$$x' = \frac{Q^2}{2k \cdot q} = \frac{x_{Bj}}{\xi} \quad z' = \frac{k \cdot k'}{k \cdot q} = \frac{z_h}{\zeta}, \quad (2.5)$$

where  $k$  and  $k'$  are the four-momenta of the incident and fragmenting partons respectively.  $\xi$  and  $\zeta$  are the usual light-cone momentum fractions, which, in the collinear configuration with massless partons are given by  $k = \xi P$  and  $P_h = \zeta k'$ . We denote by  $p_T$  (not to be confused with  $p_\perp$ ) the transverse momentum, with respect to the  $\gamma^*$  direction, of the final fragmenting parton,  $P_T = \zeta p_T$ .

The semi-inclusive DIS cross section at leading order of  $\alpha_s$ , in the QCD parton model with collinear configuration, can be written, in general, as:

$$\begin{aligned} \frac{d^5\sigma^{\ell p \rightarrow \ell h X}}{dx_{Bj} dy dz_h d^2P_T} &= \sum_{i,j} \int dx' dz' d^2p_T d\xi d\zeta \delta(x_{Bj} - \xi x') \delta(z_h - \zeta z') \delta^2(\mathbf{P}_T - \zeta \mathbf{p}_T) \\ &\quad \times f_i(\xi, Q^2) \frac{d\hat{\sigma}_{ij}}{dx' dy dz' d^2p_T} D_j^h(\zeta, Q^2). \end{aligned} \quad (2.6)$$

To first order in  $\alpha_s$  the partonic cross section is given by [8, 9]

$$\frac{d\hat{\sigma}_{ij}}{dx' dy dz' d^2p_T} = \frac{\alpha^2 e_q^2}{16\pi^2 Q^4} y L_{\mu\nu} M_{ij}^{\mu\nu} \delta\left(p_T^2 - \frac{z'}{x'}(1-x')(1-z')Q^2\right), \quad (2.7)$$

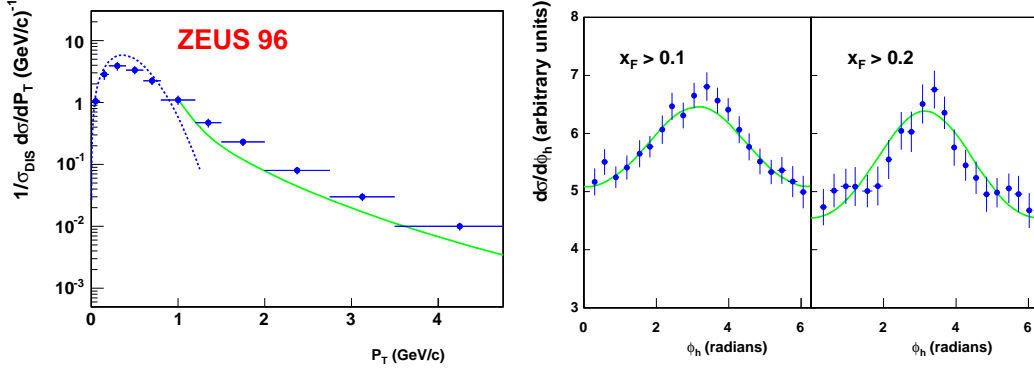
where  $ij$  denote the initial and fragmenting partons,  $ij = qq, qg, gq$ .

Large values of  $P_T$  cannot be generated by the modest amount of intrinsic motion [1]; we expect that Eq.(2.6) will dominantly describe the cross sections for the lepto-production of hadrons with  $P_T$  values above 1 (GeV/c).

Let us finally briefly consider the contributions of order  $\alpha_s^2$  (NLO),  $d\sigma_2$  in Eq. (2.2). These, for the production of large  $P_T$  hadrons, have been recently computed [10, 11], we will effectively include  $\mathcal{O}(\alpha_s^2)$  contributions in our computations by writing Eq. (2.2) as

$$d\sigma = \alpha_s^0 d\sigma_0 + \alpha_s^1 K d\sigma_1 + \dots, \quad (2.8)$$

where  $d\sigma_0$  and  $d\sigma_1$  are calculated according to Eqs. (2.3) and (2.6) respectively. The first term dominates at  $P_T \lesssim 1$  GeV/c, and the second one at  $P_T \gtrsim 1$  GeV/c.



**Figure 1:** The normalized cross section  $d\sigma/dP_T$  and  $d\sigma/d\phi_h$ : the dashed line reproduces the  $\mathcal{O}(\alpha_s^0)$  contribution, computed by taking into account the partonic transverse intrinsic motion at all orders in the  $(k_\perp/Q)$  expansion, Eq. (2.3); the solid line corresponds to the SIDIS cross section as given by LO contributions and a  $K$  factor ( $K = 1.5$ (left panel) and  $K = 6$ (right panel)) to account for NLO effects, Eqs. (2.6)–(2.8). The data are from ZEUS collaboration measurements [15] and EMC measurements [5].

### 3. Numerical Results

Fig. 1 shows our results obtained by adding, according to Eq. (2.8), the contributions of Eq. (2.3) to the contributions (computed above  $P_T = 1$  GeV/c) of Eq. (2.6). We have used

$$\langle k_\perp^2 \rangle = 0.28 \text{ (GeV)}^2 \quad \langle p_\perp^2 \rangle = 0.25 \text{ (GeV)}^2, \quad (3.1)$$

again constant and flavour independent. The  $K$  factor was fixed to be a constant, with different values according to the different  $P_T$  and  $Q^2$  ranges involved. Here and throughout the paper we have adopted the MRST01 NLO [16] set of distribution functions and the fragmentation functions by Kretzer [17] at NLO.

The dashed lines reproduce the  $\mathcal{O}(\alpha_s^0)$  contribution, computed by taking into account the partonic transverse intrinsic motion at all orders in  $(k_\perp/Q)$ , whereas the solid lines correspond to the SIDIS cross section as obtained by including LO corrections and the  $K$  to account for NLO effects. The two contributions together give a very good complementary description of the data over the full  $P_T$  domain. Unavoidably, there is a slight mismatch at the transition point,  $P_T = 1$  GeV/c, where both contributions somewhat describe the same physics, and some kind of average should be performed to avoid double counting.

In Fig. 1 one can clearly see that there present two distinct regimes. Low  $P_T$  region is mostly due to non perturbative intrinsic  $k_\perp$  dynamics, while high  $P_T$  domain is described by NLO QCD.

### 4. Predictions for forthcoming measurements

New data are expected from ongoing measurements or data analysis at HERMES, COMPASS and JLab. The COMPASS experiment at CERN collects data in  $\mu d \rightarrow \mu h^\pm X$  processes at  $p_{lab} = 160$  GeV/c. Finally, JLab collects and will collect data in the collisions of 6 and 12 GeV.

In Fig. 2, we plot the average value of  $\cos \phi_h$  as function of one variable at a time, either  $z_h$ ,  $x_{Bj}$ ,  $y$  or  $P_T$ .

Here  $\langle \cos \phi_h \rangle$  is defined as

$$\langle \cos \phi_h \rangle = \frac{\int dx_{Bj} dQ^2 dz_h d^2 P_T \cos \phi_h d^5 \sigma}{\int dx_{Bj} dQ^2 dz_h d^2 P_T d^5 \sigma}, \quad (4.1)$$

where  $d^5 \sigma$  denotes the fully differential cross section

$$d^5 \sigma \equiv \frac{d^5 \sigma^{\ell p \rightarrow \ell h X}}{dx_{Bj} dQ^2 dz_h d^2 P_T}. \quad (4.2)$$

The cross section is heavily dominated by the  $d\sigma_0$  term of Eq. (2.2), computed according to Eq. (2.3).

All the above mentioned experiments operate in the range of low  $P_T$ . We conclude that non perturbative dynamics is very important in order to understand and predict the forthcoming results.

These results depend on intrinsic momenta, both  $k_\perp$  in the partonic distributions and  $p_\perp$  in the quark fragmentation. This is obvious for quantities like  $d\sigma/dP_T$  and  $\langle \cos \phi_h \rangle$  which could not even be defined, at  $\mathcal{O}(\alpha_s^0)$ , without intrinsic motion (and pQCD corrections are negligible for the experiments we consider). However, this is also true for the differential cross-sections in  $z$ ,  $x_{Bj}$  and  $y$ : although they get contributions from intrinsic motion only at  $\mathcal{O}(P_T/Q)^2$  these contributions can be sizeable in the kinematical domains of HERMES, COMPASS and JLab.

## 5. Conclusions

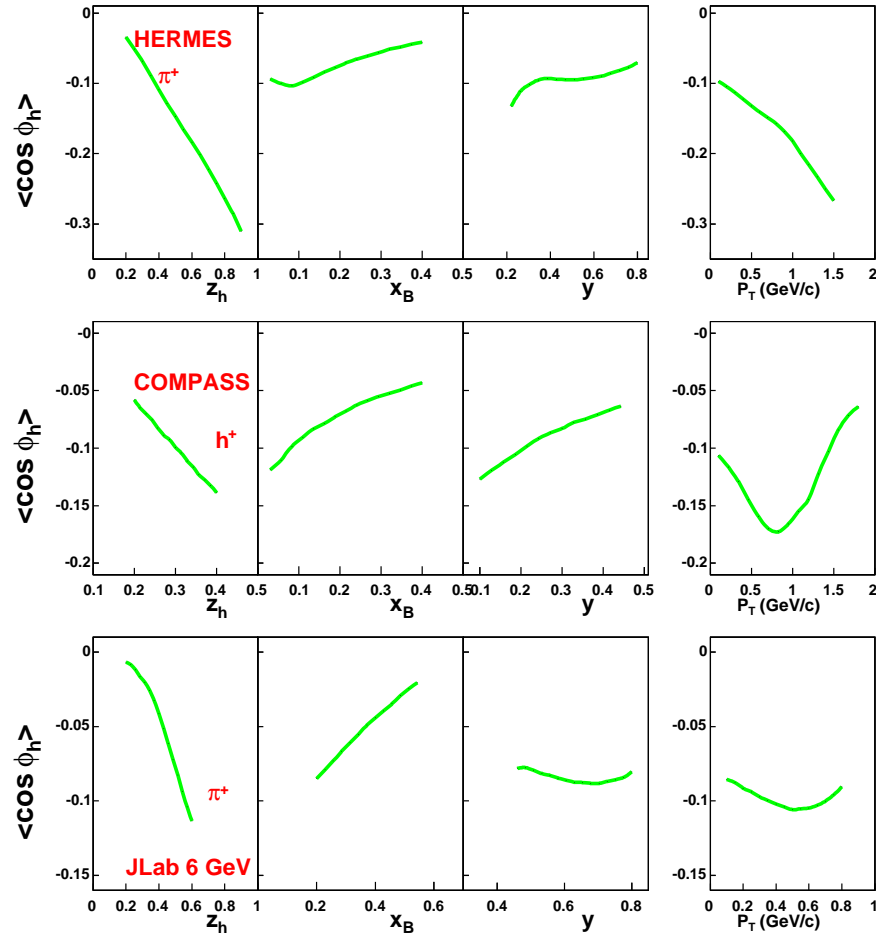
We have considered the azimuthal and  $P_T$  dependence of SIDIS data from low to large  $P_T$ ; they both cannot be explained in the simple parton model ( $\mathcal{O}(\alpha_s^0)$ ) with collinear configurations. They can originate from intrinsic motions and/or pQCD corrections. The outcome of our analysis turns out to be very simple: up to  $P_T \lesssim 1$  GeV/c the simple parton model with unintegrated distribution and fragmentation functions explains the data and leads to a good evaluation of  $\langle k_\perp \rangle$  and  $\langle p_\perp \rangle$ , while at larger  $P_T$  values,  $P_T \gtrsim 1$  GeV/c, the perturbative QCD contributions originating from hard gluonic radiation processes and elementary scattering initiated by gluons, are dominant.

Having clearly established the complementarity of the two approaches, and in particular having gained full confidence in the domain of applicability of the unintegrated parton model, we have given predictions for the cross sections and for the values of  $\langle \cos \phi_h \rangle$ , as they will soon be measured by HERMES, COMPASS and JLab collaborations, mainly in the low  $P_T$  region. These new data will be a very important tool to test our knowledge of the intrinsic partonic internal motion and on the TMD quark distribution and fragmentation functions.

## References

- [1] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin, *Phys. Rev.* **D71** (2005) 074006.
- [2] R.N. Cahn, *Phys. Lett.* **B78** (1978) 269; *Phys. Rev.* **D40** (1989) 3107.
- [3] X. Ji, J.-P. Ma and F. Yuan, *Phys. Rev.* **D71** (2005) 034005.

- [4] EMC Collaboration, J.J. Aubert *et al.*, *Phys. Lett.* **B130** (1983) 118.
- [5] EMC Collaboration, M. Arneodo *et al.*, *Z. Phys.* **C34** (1987) 277.
- [6] Fermilab E665 Collaboration, M.R. Adams *et al.*, *Phys. Rev.* **D48** (1993) 5057.
- [7] H. Georgi and H. D. Politzer, *Phys. Rev. Lett.* **40** (1978) 3.
- [8] A. Mendez, *Nucl. Phys.* **B145** (1978) 199; A. Konig, P. Kroll, *Z. Phys.* **C16** (1982) 89.
- [9] J. Chay, S.D. Ellis and W.J. Stirling, *Phys. Rev.* **D45** (1992) 46.
- [10] B.A. Kniehl, G. Kramer, M. Maniatis, *Nucl. Phys.* **B711** (2005) 345.
- [11] A. Daleo, D. de Florian and R. Sassot, *Phys. Rev.* **D71** (2005) 034013.
- [12] M. Anselmino, M. Boglione, A. Prokudin and C. Turk, “*Semi-inclusive deep inelastic scattering processes from small to large  $P_T$* ”, arXiv:hep-ph/0606286.
- [13] X. Ji, J-W. Qiu, W. Vogelsang and F. Yuan, e-Print Archive: hep-ph/0602239; *Phys. Rev.* **D73** (2006) 094017.
- [14] X. Ji, J-W. Qiu, W. Vogelsang and F. Yuan, *Phys. Lett.* **B638** (2006) 178.
- [15] M. Derrick *et al*, ZEUS Collaboration, *Z. Phys.* **C70** (1996) 1.
- [16] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *Phys. Lett.* **B531** (2002) 216.
- [17] S. Kretzer, *Phys. Rev.* **D62** (2000) 054001.



**Figure 2:** Predictions for  $\langle \cos \phi_h \rangle$  corresponding to the production of  $\pi^+$  as it will be measured by the HERMES and JLab and  $h^+$  production at COMPASS collaboration in the forthcoming future. The solid lines correspond to  $\langle \cos \phi_h \rangle$  we find by including all orders in the  $(k_\perp/Q)$  expansion. Notice that QCD corrections have no influence in this range of low  $P_T$ 's.